Name:______________________

AP Statistics HW Section 8.1/ 8.2 Binomial Distribution & Geometric Distribution:2023

Q1: A manufacturer produces a large number of toasters. From past experience, the manufacturer knows that approximately 4% are defective. In a quality control procedure, we randomly select 40 toasters for testing. We want to determine the probability that no more than one of these toasters is defective.

(a) Is a binomial distribution a reasonable probability model for the random variable X? State your reasons clearly.

(b) Determine the probability that exactly one of the toasters is defective.

(c) Define the random variable. X = _______________. Then find the mean and standard deviation for X.

(d) Find the probability that at most two of the toasters are defective. (Include enough details so that it can be understood how you arrived at your answer.)

Q2. Draw a card from a standard deck of 52 playing cards, observe the card, and replace the card within the deck. Count the number of times you draw a card in this manner until you observe a jack. Is a binomial distribution a reasonable probability model for the random variable X? State your reasons clearly.

Q3: In parts (a) and (b), indicate whether a geometric distribution is a reasonable probability model for the random variable X. Give your reasons in each case.

(a) Suppose that one of every 100 people in a certain community is infected with HIV. You want to identify an HIV-positive person to include in a study of an experimental new drug. How many individuals would you expect to have to interview in order to find the first person who is HIV-positive?

(b) In high-profile discrimination court cases in the past, 76% of prospective jurors have been found eligible to serve on juries (that is, no objection by either the prosecution or the defense). We have 25 people in the pool of potential jurors, and we want to know if we will be successful in finding 12 people to serve on the jury from the pool. Specifically, we want to determine the probability that the 12th acceptable juror is found by the time that the 25th prospective juror is interrogated.

Q4: A fair coin is flipped 20 times.

(a) Determine the probability that the coin comes up tails exactly 15 times.

(b) Find the probability that the coin comes up tails at least 15 times. (Include enough details so that it can be understood how you arrived at your answer.)

(c) Find the mean and standard deviation for the random variable X in this coin-flipping problem.

(d) Find the probability that X takes a value within 2 standard deviations of its mean.

Q5: When a computerized generator is used to generate random digits, the probability that any particular digit in the set {0, 1, 2, . . . , 9} is generated on any individual trial is 1/10 = 0.1. Suppose that we are generating digits one at a time and are interested in tracking occurrences of the digit 0.

(a) Determine the probability that the first 0 occurs as the fifth random digit generated.
(b) How many random digits would you expect to have to generate in order to observe the first 0?
(c) Construct a probability distribution histogram for X = 1 through X = 5. Use the grid provided.

Q6: There is a probability of 0.08 that a vaccine will cause a certain side effect. Suppose that a number of patients are inoculated with the vaccine. We are interested in the number of patients vaccinated until the first side effect is observed.

i) Define the random variable of interest. X = ________________________________

ii) Verify that this describes a geometric setting.

iii) Find the probability that exactly 5 patients must be vaccinated in order to observe the first side effect.

iv) Construct a probability distribution table for X (up through X = 5).

v) How many patients would you expect to have to vaccinate in order to observe the first side effect?

vi) What is the probability that the number of patients vaccinated until the first side effect is observed is at most 5?

Q7: The makers of a diet cola claim that its taste is indistinguishable from the full calorie version of the same cola. To investigate, an AP Statistics student named Emily prepared small samples of each type of soda in identical cups. Then, she had volunteers taste each cola in a random order and try to identify which was the diet cola and which was the regular cola. Overall, 23 of the 30 subjects made the correct identification. If we assume that the volunteers really couldn't tell the difference, then each one was guessing with a 1/2 chance of being correct. Let X = the number of volunteers who correctly identify the colas.

Problem:
(a) Explain why X is a binomial random variable.

(b) Find the mean and the standard deviation of X. Interpret each value in context.

(c) Of the 30 volunteers, 23 made correct identifications. Does this give convincing evidence that the volunteers can taste the difference between the diet and regular colas

Q8:

Suppose we have a random variable X where $P(X = k) = \binom{15}{k}(.29)^k(.71)^{15-k}$ for $k = 0, 1, \ldots, 15$. What is the mean of X?

(A) 0.29
(B) 0.71
(C) 4.35
(D) 10.65
(E) None of the above